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American Nautical Almanac Office ; Annals of Harvard College Observatory, vol. xxxii., Part I. (Investigations in Astronomical Photography, by W. H. Pickering), presented by the Observatory ; a sixteenth century celestial globe, on bronze tripod stand, presented by the Earl of Crawford ; three large transparencies of portions of the Moon, from negatives made with the equatorial coudé of the Paris Observatory, presented by MM. Lœwy and Puiseux.

On the Rotation and Mechanical State of the Sun.

By R. A. Sampson, M.A.

(*Abstract.*) *

The author's aim is to consider with the simplest sufficient hypotheses the mathematical theory of the mechanical state and the relative motions of the parts of the Sun.

Starting from the equations of equilibrium of a gas, it appears that equilibrium is a state which will only exist either (1) when the temperature is maintained a function of the density, or (2) when all quantities are geometrically functions of only one parameter.

In the former case it is shown under what condition a body of gas will cohere in finite volume, and that if this condition is unfulfilled the density can nowhere be finite unless the total mass be infinite.

The study of the second case begins with the law of distribution of temperature. Conduction is assumed to be inoperative until instability producing convection currents is proved to exist, so that the agent of distribution must be taken to be radiation. The author shows that if three hypotheses be made –

(1) Each isolated small body of gas radiates heat at a rate proportional to its mass and to its absolute temperature conjointly.

(2) Each small body of gas absorbs heat proportionally to its mass, and to the quantity of heat that penetrates it, conjointly.

(3) The entire radiations from any element of a sheet of matter pursue the same course (normal to the wave fronts) without interference ; then the temperature (θ) satisfies a partial differential equation, which yields the set of solutions—

$$\theta_n = \alpha_n \, exp\left(-\frac{2\varpi\mu^2{}_n}{\mu^2_n + \varpi'^2}\,\frac{t}{c}\right)\cos\mu_n m,$$

where μ_n is one of the roots of

$$\mu \tan \mu \mathrm{M} = \varpi',$$

α is arbitrary, c, ϖ, ϖ' are certain constants, M is the total mass, and m is the mass within a certain surface passing through the

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* For the complete paper, vide *Memoirs*, vol. li.

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point considered. This applies to all arrangements in which the quantities are functions of but one parameter. Hence the temperature of a body cooling under radiation may be analysed into a number of elements, each of which cools at a definite rate; ultimately all others will disappear relatively to that of slowest rate.

Considering the equilibrium of a spherical body of gas under this law of temperature, it is shown that when the body is very great, the distribution of mass, density, and temperature is the following:—

r/a.	m/M	ρ/ρ_0.	θ/θ_0.
·0	·000	1·000	1·000
·1	·001	1·000	1·000
·2	·005	1·000	1·000
·3	·017	1·000	1·000
·4	·051	1·002	·997
·5	·079	1·008	·992
·6	·138	1·024	·977
·7	·223	1·064	·939
·8	·343	1·164	·858
·9	·520	1·461	·685
1·0	1·000	∞	0

Since the density increases outwards the distribution may be proved to be unstable, but the margin of instability being generally very small, the disturbances thence produced are not enough to change substantially the above distribution, except in the neighbourhood of the surface, where, if necessary, it may be supposed to give place to that of convective equilibrium.

If this theory be taken to apply to the Sun, we have a body affected throughout by convective currents, but substantially at uniform density and temperature, except near the surface, where this arrangement gives place to a rapid fall of temperature and density, accompanied by violent disturbance. This region of superficial violent disturbance appears as the main radiator, and it is shown numerically that this is a reasonable supposition. The main body, which is subject to less violent disturbances, radiates at a much lower rate, thence producing an effect of relative darkness.

Considering the rotation of the Sun, it appears that the data are not adequate to prove more than merely the sufficiency of an assigned cause. Starting from the equations of motion of a viscous gas, it is shown that Dunér's observations of daily angular rotation for 60° N. and S. lat. are consistent with the supposition of a velocity increasing inwards, and expressed by the formula

$$\omega = \omega_0 + r^{n-2}(\Omega - \omega_0),$$

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where Ω is the observed surface rotation at any latitude, ω_0 is a constant derived from the calculations, and n is nearly equal to $2\frac{1}{4}$; taking $n=2\frac{1}{4}$, ω_0 is about one half the equatorial value of Ω. The data being indeterminative, the observations are also consistent with the above expression supplemented by an infinite arbitrary series; but it is shown that to attach a finite value to any term of this series implies that a finite part of the angular motion increases inwards at least as fast as r^{-2}, that is to say at much greater rate than the motion at the surface decreases towards the poles.

Finally it is shown that if the atmosphere follow the law of convective equilibrium, the associated surface motion in latitude is a slight drift towards the poles.

The Wilsonian Theory and the Stonyhurst Drawings of Sun Spots.

By the Rev. Walter Sidgreaves, S.J.

The question of the cavernous nature of a Sun spot has been recently brought prominently before the attention of Solar Physicists by the Rev. Mr. Howlett in a paper read at the last December meeting of the Royal Astronomical Society. His handsome present to the library of the Society is in evidence of the care and labour devoted to the subject, and the long series of large-size drawings contained in those eight volumes, covering a period of thirty-five years, seem to place his verdict against the Wilsonian theory beyond the reach of contradiction, and to show that it stands in no need of a confirming witness.

The Stonyhurst drawings extend over considerably less than half the years of Mr. Howlett's work, and are made to a scale, in diameter, only the one-third of his. They are therefore of much less weight, both in number and size. They were not commenced nor carried on with the direct object of testing the Wilsonian theory; but this was never lost sight of, and occasionally, when a spot was near the limb, a special drawing was made of it from an enlarged image, after the usual full surface drawing of 10½ inches diameter was complete. The smaller drawing of the spot on such occasions was never retouched after its examination under the greater magnification; and the uniform agreement between the pairs confirms the rule that whatever is seen satisfactorily with less magnification is never contradicted by an enlarged image.

To form a safe estimate of the value of the smaller drawings for the subject of discussion, it should be remembered that in all cases the outlines of the umbra and penumbra were traced with a finely-pointed pencil over the image upon the paper, and that no drawing was considered complete until the draughtsman was satisfied that his picture faithfully represented what he saw. Hand-drawings made in this way have a value hardly inferior to